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AUTHOR	Woldbeck, Tanya
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ABSTRACT

This paper outlines two types of discriminant analysis, predictive discriminant analysis (PDA) and descriptive discriminant analysis (DDA). Important differences between PDA and DDA are introduced and discussed using a heuristic data set, specifically indicating the portions of the Statistical Package for the Social Sciences (SPSS) output relevant to each type of discriminant analysis. The importance of knowing distinguishing features of PDA and DDA is highlighted using an example of stepwise discriminant analysis. Data are taken from the dataset of K. Holzinger and F. Swineford (1939) for 301 seventh and eighth graders. Two appendixes illustrate computer programs for discriminant analysis. (Contains 9 tables and 29 references.) (Author/SLD)

Running Head: DISCRIMINANT ANALYSIS

Two Types of Discriminant Analysis: NOT Six of One, Half a Dozen of Another

Tanya Woldbeck

Texas A&M University 77843-4225

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Abstract

The present paper outlines two types of discriminant analysis, predictive discriminant analysis (PDA) and descriptive discriminant analysis (DDA). Important differences between PDA and DDA are introduced and discussed using a heuristic data set, specifically indicating the potions of the SPSS output relevant to each of the types of discriminant analysis. The importance of knowing the distinguishing features of PDA and DDA is highlighted using an example of stepwise discriminant analysis.

Two Types of Discriminant Analysis: NOT Six of One and Half a Dozen of the Other

In areas of behavioral science such as psychology and education, researchers tend to look at how a number of variables (e.g., SES, IQ, and parent involvement) affect a specific outcome (e.g., academic achievement). It is rare in these areas of research to find efforts looking at only one variable at a time. Huberty (1994) reminds us that

> empirical research in nearly every discipline is rarely confined to the study of a single response variable, a characteristic or attribute or trait on which the researcher obtains scores or responses for a collection of units. Data sets typically involve measures on a number of variables, and it may be desirable to consider...all the variables *simultaneously*. (pp. 26-27)

This model of reality makes it important for researchers to understand the concepts of multivariate statistics (Thompson, 1994b). Thompson (1994b) argued that multivariate statistics "limit the inflation of Type I 'experimentwise' error rates" (p. 9). A lower Type I error rate helps ensure correct conclusions, which is important to sustaining the life of research efforts (Thompson, 1994b). More importantly, multivariate analyses also "best honor the reality to which the researcher is purportedly trying to generalize" (Thompson, 1994b, p. 12), that is, a reality in which most effects are multiply caused and most causes have multiple effects.

Multivariate analyses can be conceptualized in a number of ways. While Knapp (1978), in his discussion of the general linear model, did conclude that "all of the commonly encountered parametric tests of significance can be treated as special cases of canonical correlation analysis" (p. 410), each multivariate technique is unique in its own way. It is important for researchers to understand the similarities and the differences among multivariate analyses. Techniques such as factor analysis are used when interested in clustering variables, or, in other words, when we want to study individual differences on a single group of variables (Huberty, 1994). Klecka (1980) describes a technique, the focus of this paper, which does not look at individual cases, but at data cases that comes from two or more mutually exclusive groups: discriminant analysis.

While univariate techniques are still the most popular analyses, the use of multivariate techniques is on the rise in research journals (Elmore & Woehlke, 1988). For example, Emmons, Stallings and Layne (1990) studied 16 years of research reports in three journals, and found that

> the multivariate characteristic of the social science research environment with its many confounding or intervening variables has been addressed through the trend toward increased use of multivariate analysis of variance and covariance, multiple regression, and multiple correlation. (p. 14)

Similarly, Grimm and Yarnold (1995) recently noted that, "in the last 20 years, the use of multivariate statistics has become commonplace. Indeed, it is difficult to find empirically based articles that do not use one or another multivariate analysis" (p. vii).

One unfortunate impetus for increased use of multivariate methods is the new point-and-click microcomputer software environment in which researchers now work. Computer point-and-click software makes blind reliance of its use very easy. Many researchers tend to grab output and analyze data haphazardly, with no real understanding of these sophisticated multivariate techniques; discriminant analysis is no stranger to this phenomenon.

The present paper explains some of the basic principles and concepts underlying discriminant analysis by differentiating two types of discriminant analysis (DA), predictive discriminant analysis (PDA) and descriptive discriminant analysis (DDA). These differences will be clarified and concretely illustrated using a heuristic data set taken from Holzinger and Swineford (1939). Holzinger and Swineford (1939) collected data on 301 seventh and eighth graders. The data represent scores on a number of performance tests ranging from reading comprehension to mathematical reasoning.

The importance of understanding the differences between PDA and DDA (see Huberty, 1994, and Thompson, 1995a) will be highlighted by applying the distinctions to specific research questions and situations. The paper cautions against blind reliance on statistical software packages. The package illustrated in this paper is SPSS, although these issues apply to other software as well.

Discriminant analysis is one multivariate method available "[w]hen groups of units are known in advance and the purpose of the research is either to describe group differences or to predict group membership on the basis of response variable measures" (Huberty, 1994, p. 28). DA is a procedure which "identifies boundaries between groups of objects, the boundaries being defined in terms of those variable characteristics which distinguish or discriminate the objects..." (Kachigan, 1982, p. 216).

The units of analysis in discriminant analysis, whether we are studying people, animals or political parties, need to be from two or more mutually exclusive groups, and each case needs to have corresponding values on some set of response variables, which are measured on an interval or ratio level of measurement (Klecka, 1980). The grouping variable, on the other hand, is categorical in nature. It is DA's ability to look at this nominal variable that make it desirable in behavioral science research. Multiple regression is inappropriate in that the dependent variable in this prediction model needs to be continuous in nature. As in other analyses, the response variables are combined in a linear fashion to describe groups or to predict group membership (Huberty, 1994).

The use of DA carries with it some assumptions. These assumptions are fully outlined in Klecka (1980) and Huberty (1994). One assumption is that each group is drawn from a population with multivariate normal distributions of the response variables, and no response variable can be a linear combination of any other variables. For example, it would be impermissible to use dollars earned per month last year in an equation along with dollars earned last year and number of months worked last year. While it is beyond the scope of the current paper to discuss the intricacies of parameter estimation in DA, the interested reader is encouraged to refer to Klecka and to Huberty for a detailed summary. Ashcraft (1998) also summarizes these issues and provides a nice short SPSS for Windows program that implements a graphical test of multivariate normality; the program is reproduced here in Appendix A. The rest of this paper focuses on describing and contrasting the two types of discriminant analysis, PDA and DDA.

To repeat, their are two separate reasons why a researcher would use DA. One is to describe group differences, and one is to predict group membership. While both fall under the umbrella of discriminant analysis, the two types of DA differ in focus, analysis, and interpretation. The two analyses also differ historically. Huberty (1994) noted that PDA (and the associated linear classification function (LCF)) was conceptualized first. DDA (and the associated linear discriminant function (LDF)) came later in order to explain effects found in MANOVA. LDF's and DDA's relation to MANOVA will be covered later.

Prior to discussing PDA, however, an important note needs mentioning. Table 1 includes a sample output from an SPSS run. The computer was given a number of variables from Holzinger and Swineford (1939), and told to run a discriminant analysis (sample syntax can be found in Appendix B). Note that the syntax does not indicate "PDA" or "DDA". SPSS gives you everything you need to do either PDA or DDA, but does not differentiate the relevant aspects of the results as regards these analyses. You, the researcher are left to discern the relevant information from the irrelevant information given your research question. This is a good example of the need to fully understand the analyses one asks of computers, and what information needs to be focused on depending on the research question. Of course, since many researchers blindly use computers, and have blind faith in software, many researchers not seeing SPSS distinguish PDA from DDA, wrongly presume that all printed results are relevant to both analyses.

Insert Table 1 about here

Predictive Discriminant Analysis

As previously indicated, PDA is used when researchers wish to predict a data case's group membership based on responses on a number of variables of interest. While data cases could be things, people, or animals, this paper, using data on seventh and eighth graders, will refer to data cases as people. Klecka (1980) describes PDA as "the process by which a decision is made that a specific case 'belongs to' or most closely resembles' one particular group" (p. 42). A research question posed for the Holzinger and Swineford (1939) data, therefore, might be, "Given student responses/scores on performance measures, can we predict their grade placement?"

In PDA, response variables serve as the predictor, or independent variables, and the grouping variable is the dependent or criterion variable. Data are collected on a number of response measures, and these variables are used to form a classification "rule" (also called a "function," "factor," or an "equation", mainly to confuse the graduate students) which will help predict membership for a unit (person). According to Huberty (1994), a classification rule can take on various forms: "...that of a composite of the predictor measures;...that of a probability of population membership;... [and] that of a distance between two points" (p. 40).

With regard to distance, assume we have three measures, and consider two people's scores on these three measures as points in a three-dimensional space. We can look at three types of distances in this space "(1) unit to unit... (2) centroid to centroid,[see Ashcraft, 1998, for an explanation] ...and (3) unit to centroid" (Huberty, 1994, p. 44). A unit is one person's vector of scores, and the centroid is the Cartesian coordinate defined by the means across all the response variables. A decision on group membership is based on which centroid to which a unit falls closest.

Group prediction can also be accomplished through the use of a linear classification function, or LCF. An LCF is a linear combination of the measures of interest, and there are as many LCF's as there are groups to be predicted (Huberty, Wisenbaker, & Smith, 1987). As in other linear composites of response variables (i.e., linear regression), weights are applied to each of the independent predictor variables, and "[t]he weights in these composites are products of sample mean response variable vectors and the inverse of the pooled within-groups covariance matrix" (Huberty et al., 1987, p. 309). LCFs are used only when the population covariances are thought to be equal; "otherwise, *quadratic* classification functions (QCFs) may be used" (Huberty & Lowman, 1997, p. 766). In other words, the phrases "linear" rule or "linear" weights merely mean that the weights are computed using the *pooled* variance-covariance matrix on the response variables, while the phrases "quadratic" rule or weights merely mean that the weights are computed using the *separate* variance-covariance matrix on the response variables for each group in the analysis (see Young, 1993, for more details).

In PDA, cases receive an LCF score on each LCF by applying the LCF weights to the response variables. Thus, since the number of LCF's equals the number of groups, each case has a number of LCF scores equal to the number of groups. In PDA, each case is assigned to the group corresponding to the LCF on which the case has the highest score (e.g., if there are three groups, and case "A" had LCF scores of 1.97, .05, and .35, respectively, that case would be predicted to belong to group number one).

Classification into a certain group can also be discussed in terms of probabilities. One important probability to look at is prior probabilities. Huberty (1994) points out that the "goodness [of a classification rule]...depends on the size (and representativeness) of the...original samples..." (p. 47). If one group comes from a large population, then it is more likely that a sample will be drawn from it. Huberty (1994) likens priors to a base rate. Huberty and Lowman (1997) reported that "[t]here are three types of prior probabilities...: equal priors, proportional priors [based on the sample size]..., and specified priors [set by the researcher ahead of time]" (p. 765). All of these options are available in SPSS. The data in this paper were run using proportional priors, which presumes that in the population the groups are not equal in size.

Once prior probabilities are taken into consideration, unit analyses can be done to determine posterior probabilities to decide upon group membership. The basic premise behind posterior probabilities is to determine the probability of group membership given one's scores on the response variables. The unit is placed in the group which yields the highest posterior probability (Huberty, 1994). As Huberty (1994) points out, "SPSS DISCRIMINANT yields only the two largest [posterior probability] values" (p. 74). However, since probabilities always sum to 1.0, if there are three groups, the missing probability can be easily calculated by subtracting the two printed probabilities from 1.0. These values can be found in Table 2, and were also removed from the original printout in Table 1.

Insert Table 2 about here

Insight into classification can be expanded by analyzing “fence riders” and “outliers”. Outliers are determined by looking at the typicality probabilities. Typicality probabilities ask the question, given placement in group 2, "[t]o what extent does that unit 'look' like the typical member of Group 2" (Huberty & Lowman, 1997, p. 769)? Huberty (1994) cautions that an outlier "may not belong to any group..." (p. 76). Fence riders are those units which split their posterior probabilities evenly between two groups. We would have less confidence in our classification results with too many fence riders, as "[f]or each of these units, it is nearly as probable that they belong to one group as it is that they belong to another group" (Huberty, 1994).

In addition to looking at individual units, it is important to assess PDA group results in a classification table, an example of which can be found in Table 3. Table 3 shows a classification table based on the Holzinger and Swineford (1939) data, predicting grade level from student responses on 24 measures. The percentages in the table represent our success at prediction, or our hit rate. The hit rate is <u>the</u> focus of the PDA analysis. It answers the question, how well did we predict? This question can be answered in terms of overall classification and classification by group.

Insert Table 3 about here

Methods of assessing predictive accuracy are discussed quite thoroughly in Huberty, Wisenbaker and Smith (1987). These authors describe three types of hit rates, which are also discussed in Huberty (1994). “Optimal” hit rates are "based on known parameters...applied to the population" (Huberty et al., 1987, p. 310). An “actual” hit rate is a rule which is applied to future samples, and the “expected” hit rate is the hit rate when a rule is applied to all possible samples (Huberty et al., 1987).

These probabilities, or hit rates, are estimated through an “internal” analysis or an “external” analysis. An internal analysis uses the data at hand to obtain a classification rule. This rule is then applied to the data and results are given in a classification table, as mentioned above (Huberty & Lowman, 1997). Huberty (1994) and Huberty and Wisenbaker (1992) emphasize that internal analyses provide positively biased hit rates, and that a better technique would be an external analysis, where classification rules based on one set of data are applied to a different set of data for accuracy assessment. Two external analyses, outlined in Huberty (1994) and Huberty and Lowman (1997), are the holdout method and the leave-one-out (L-O-O) method. These methods use future data to assess predictive accuracy. Both external methods are available in SPSS, however, the L-O-O method must be added directly to the syntax. Subcommands are available in Huberty and Lowman (1997).

Descriptive Discriminant Analysis

Describing group differences is the job of what has been called descriptive discriminant analysis (DDA). We are not concerned with predicting a unit's membership in a group; this is now taken as known. Instead, DDA is used to describe the effects of group membership, for example, to further explain the effects found in a MANOVA analysis. This paper assumes that the reader is familiar with MANOVA. In brief, classical MANOVA uncovers overall or "omnibus" differences between two groups. If statistically significant omnibus effects are detected, DDA is then used to detect (a) which groups differ and (b) on which response variable the groups differ. In a distance sense, different groups have centroids that are farther apart. DDA teases out thee differences. Said in another way, MANOVA assesses effects while DDA describes them (Huberty, 1994). Parenthetically, it might be noted that a one-way MANOVA yields exactly the same results as DDA as regards the omnibus test of groups differences on the response variables, but the DDA also simultaneously provides the more detailed information about (a) which groups differ and (b) on which response variables they differ. Thus, the prudent researcher with a one-way MANOVA problem would never do a MANOVA followed by a DDA, and would always move straight away to DDA.

In the case of DDA, the response variables are the criteria or the dependent variables. This is opposite from the logic in PDA. In DDA, we are now focusing on the groups and how they differ on a set of response variables, which are combined to compute a linear discriminant function (LDF). The response variables are combined in a such a way that the response variable differences between the groups is maximized. One difference between PDA and DDA involves the number "rules" ("functions", "factors", or "equations"); in the PDA the number of LCF rules (i.e., sets of weights) equals the number of groups, while in DDA the number of LDF rules equals the smaller of (a) the number of response variables or (b) the number of groups minus 1.

The focus in DDA, therefore, is on maximum group separation. What is the best way to describe the differences between groups of data cases? Here the grouping variable is the independent variable (Huberty & Barton, 1989). In this context DDA is considered a post hoc analysis after statistically significant results have been realized in a MANOVA analysis (Huberty, 1984). A DDA research question, then, for Holzinger and Swineford (1939) may be, "how can we best describe the response variable differences between seventh and eighth grade students?".

The SPSS LDF weights for this analysis can be found under the heading "unstandardized canonical discriminant function coefficients" (Huberty & Lowman, 1997). The relevant output (taken from the output from Table 1) can be found for the DDA in Table 4.

Insert Table 4 about here

While there are more LDF's available than LCF's, we may not be interested in all of them. We are only interested in those which describe noteworthy differences between groups. Huberty and Lowman (1997) remind us that "[t]o best describe the grouping variable effects, it is important to focus only on 'relevant' LDF's" (p. 774). Huberty (1994) and Huberty and Lowman (1997) described three ways to assess relevance: LDF plots, statistical tests, and looking at the "correlations between linear composite scores and scores on the individual variables in the composites" (Huberty, 1994, p. 209), or structure r's (see Thompson, 1997).

Statistical tests use the Wilks' Λ (with a corresponding F and p value) to test the null hypothesis that there is no dimension ("rule", etc.) which adequately separates the groups. These tests are available in SPSS and an example is found in Table 2. Another strategy is to plot the centroids of the LDF scores (called "discriminant scores", akin to the ubiquitous regression "yhat" scores). This allows to view the data in physical space to view the actual separation of the groups.

Analyzing the structure r's uses the LDF as a principal component of sorts. While there are other ways of finding the values of the structure r's, Huberty (1994) recommends using the within-group or error structure r's. Huberty states that "[t]he idea behind the use of the structure r's is that the variables that share the most variation with a given LDF should define what the LDF represents" (p. 209).

Stepwise Analysis in PDA and DDA (Both are Bad)

While multivariate methods help us look at a number of variables at once, it is often necessary to identify the variable or set of variables which do the best job at predicting a certain outcome or describing groups differences. Thus, Cohen (1994) claimed that "we should be studying few independent variables and even fewer dependent variables" (p. 1304). Cohen went on to argue that studying too many variables is redundant, and, in this author's opinion, stretches the average Ph.D. student life to 10 years plus. When we do research, the key word which drives our efforts in research and statistical analyses is parsimony. Huberty (1989) offers the following definition: "The notion of parsimony in explanation or in description or in model building is 'scientific'" (p. 45). A goal of research should be to explain the most with the least.

Let us now turn our discussion to variable ordering and selection. In the case of both

DDA and PDA, "there is a theoretical reason for retaining only 'worthwhile' variables or deleting 'worthless' variables" (Huberty, 1994, p. 227). This concept is especially crucial to PDA, as an increase in variables may actually decrease our hit rate. Increasing the number of response variables in DDA, on the other hand, can never attenuate the uncorrected effect size (Huberty, 1994).

There are a number of ways to look at variable importance and order. However, one gross misuse of this kind is stepwise statistical analysis. Thompson (1994c) includes the use of stepwise procedures as one of seven common mistakes made in doctoral dissertations. Despite continued caution against its use (Huberty, 1989; Snyder, 1991; Thompson, 1995b), "it is quite common to find the use of 'stepwise analyses' reported in empirically based journal articles" (Huberty, 1994, p. 261).

Stepwise analysis is a quick and very dirty alternative in most statistical software packages, and blind reliance on its use has led to erroneous conclusions in the literature. Essentially, forward stepwise logic is to enter a response variable into the prediction equation or discriminating equation one at a time, and based on a criterion, a decision is made to keep or remove the variable at each step. Various statistical criteria can be used in DDA, but Wilks' lambda is often used. Wilks' lambda is an inverse statistic, which means that the lower the value the better.

The steps continue until all of the variables have been considered, or until adding (or deleting) anymore will not appreciable alter the outcome. For each lambda value there is a corresponding F-statistic, which provides a test for statistical significance (Huberty, 1989). The use of statistical significance testing has been hotly debated in the literature (Carver, 1993; Snyder

& Lawson, 1993), and researchers have been cautioned against the mindless use of these tests (Cohen, 1994).

There are mainly three myths that naive researchers incorrectly believe are true of stepwise discriminant analysis. One common misconception is that stepwise procedures produce replicable results. The reason for this phenomena is that "stepwise methods tend to capitalize outrageously on sampling error" (Thompson, 1995b, p. 532). A second and related myth is that stepwise programs correctly select the best subset of variables. A final and pernicious myth is that statistical software packages are always correct, and that we should blindly accept package outputs as being correct. In fact, SPSS and similar packages utilize the wrong degrees of freedom in the calculations of stepwise statistical tests (Huberty, 1989; Snyder, 1991; Thompson, 1995b). A big problem with user-friendly statistical software packages is that "such convenient accessibility to multivariate output has led to some misuse and misinterpretation of computer printout information" (Huberty & Barton, 1989, p. 165). The only task that stepwise methods may do well is "correctly identify the best single predictor" (Thompson, 1995b, p. 532), and this is true only for the DDA case. An elaboration of these myths will be made concrete, again, with the Holzinger and Swineford (1939) data set.

<u>Stepwise Procedures Capitalize on Sampling Error</u>

"Sampling error is variability in sample data that is unique to the given sample" (Thompson, 1995b, p. 532), and therefore, makes replicability of results in future samples unlikely. In SPSS, Wilks' Λ can be used for variable selection in discriminant analysis. The F-statistic forms the basis for decision-making. Rencher and Larson (1980) explained that, "At each stage the variable with the largest F-to -enter is added to the set of variables if its F-value is larger than a specified threshold" (p. 350).

It should be noted, however, that variables are entered even if their F-values are just barely larger than other variables. Thompson (1995b) explained, "It is entirely possible that this infinitesimal advantage of [one variable] over another variable is sampling error" (p. 532). If a variable is selected by mistake very early on, this will throw off the selection of subsequent variables, as the entry decisions are conditional on all previous selection decisions (Snyder, 1991).

Rencher and Larson (1980) take this argument a step further, and claim that bias in Wilks' Λ and the instability of using statistical significance testing as a stopping rule may lead to problems such as:

> Inclusion of too many variables in the subset. If the significance level shown on a computer output... is used as an informal stopping rule, some variables will likely be included which do not contribute to separation of groups. A subset chosen with significance levels as guidelines will not likely be stable... (p. 350)

These authors go on to argue that the computer may very well select a completely spurious subset of variables, which is not detected if the researcher goes by F-values alone. This will be further discussed in the next section.

Stepwise Procedures Do Not Necessarily Choose the Best Subset of Variables

Probably the most common misconception among applied researchers is the wrong-headed belief that stepwise methods identify the best variable set of a given size. Stepwise methods do not do this. Thus, Snyder (1991) reminds us that, "Selecting the next-best predictor for each of q steps is not the same as selecting the optimal predictor variable set of size q" (p. 101). The problem can be made concrete by considering an example given by Thompson (1995b) using the analogy of a five-member basketball team. Choosing the best five individual players does not necessarily ensure that one has chosen the best overall team. There may be another team of five which, and potentially even not made up of any of the best players, work better as a team. This phenomenon is illustrated using the Holzinger and Swineford (1939) data in the following heuristic example.

The first step in the illustration was to tell the computer to perform a stepwise discriminant analysis (the syntax file is available in Appendix B). F-to-enter and F-to-remove parameters were set in order to limit the number of variables chosen to four. Table 5 shows a reproduced table from the output file indicating the variables chosen as the "best subset" were T10, T19, T3, and T17. The corresponding Wilks' Λ, F-statistics, degrees of freedom, and p levels are shown. Note that the final Λ value is .805. The naive researcher would take these results, do a couple of flips, and claim golden results. Manipulation of the data, however, reveals serious problems.

Insert Table 5 about here

The next step was to systematically enter different variables available from the data set into a non-stepwise discriminant analysis. The results, while the process was somewhat tedious, were interesting. Table 6 shows that entering variables T3, T10, T12, and T19 result in the same Λ value reached in the stepwise method. While a lower Λ value would have been ideal for illustrative purposes, an interesting point can be made regarding practical problems which arise with stepwise methodology.

Insert Table 6 about here

Indeed, it is entirely possible that a different set of four predictors might have yielded a smaller (i.e., better) Λ value. Note that the variables in Table 6 differ from those in Table 5. In Table 6, T12 substituted for T17.

T12 is a variable representing a score on a test called "speeded counting of dots in shape." T17 is labeled "memory of object-number association targets." For illustrative purposes, let us presume that T17, the variable chosen by the stepwise procedure takes 30 minutes to administer, while T12 takes only 5 minutes. This is an important difference if testing fatigue is an issue. However, without this close scrutiny, a researcher may choose T17 over T12 solely based on SPSS results. This example strongly argues against blind interpretation of computer results. It has become far too easy for researchers to stop thinking, however, and what is easy is often inappropriate.

Software Packages Use the Wrong Degrees of Freedom

While it may seem that this paper has focused on the ills of computer software, this is entirely not the case. Researchers, however, need to be aware that certain software mistakes are also evident, and need to be avoided. In stepwise analyses, statistical packages, such as SPSS, consistently use the wrong value for the degrees of freedom in the F-statistic. Thompson (1995b) describes degrees of freedom as follows:

> Degrees of freedom in statistical analyses reflect the number of unique pieces of

information present for a given research situation. These degrees of freedom constrain the number of inquiries we may direct at our data and are the currency we spend in analysis. (p. 526)

In typical programs, 1 degree of freedom explained is charged at step one of a stepwise analysis. In reality, however, all of the variables are being consulted for that given step. At step two, k-1 variables are consulted, but only two degrees of freedom explained are charged. Charging too little for the analysis results in a positively biased F-value. The degrees of freedom explained for each step should be the total number of variables since every variable is examined to decide which one to ultimately enter. And if the degrees of freedom for the numerator are wrong, then of course so are the degrees of freedom unexplained. <u>Both</u> these wrong degrees of freedom bias the calculated <u>F</u> statistic is the same direction: positively. Results of the computer package tests can be tremendously different from those employing the correct degrees of freedom. An example using the Holzinger and Swineford (1939) data is included in Table 7.

Insert Table 7 about here

The formula at the top of Table 8, $F = [(1 - \Lambda) / \Lambda]*$ (df error / df effect) is used to recalculate the F-statistic. This is done for two reasons. One reason is to show the reader that the wrong degrees of freedom can result in spuriously high F-values. The other reason is to show the reader a way to <u>correct</u> the computer's mistake. In the first calculation, the F-statistic is done the wrong way (i.e., the way SPSS would calculate it). Note that the F-value of 41.408 is almost equal to the F-value calculated and found in Table 6, 41.360. This was done to check the formula. Underneath, the F-value is recalculated using the correct degrees of freedom. The new value, 0.0105347, is quite different, and would not reach statistical significance, since an F less than 1.0 can not be statistically significant even at infinite degrees of freedom. "This statistical welfare system may cause us to radically overestimate the atypicality of our results" (Thompson, 1995b, p. 527), and again hurt any chances for replication in the future.

Double-Idiot-Syndrome: Using Stepwise Methodology in PDA

The final stake to drive into the heart of stepwise methodology highlights, again, the differences between PDA ad DDA, outlined earlier. To review, Klecka (1980) describes the DDA as follows:

> A researcher is engaged in interpretation when studying the ways in which groups differ-that is, is one able to 'discriminate' between the groups on the basis of some set of characteristics, how well do they discriminate, and which characteristics are the most powerful discriminators? (p. 9)

The last part of the question describes the focus of stepwise DDA. Which variables discriminate the groups (i.e., describe group differences) the best? To answer these questions, stepwise methodology looks at Wilks' Λ and the F-statistic. These values are used to decide which variables " do the best job".

Remember, however, that PDA is not concerned with group differences, but solely and exclusively with classification. The focus is on how well the discriminant function can predict group membership. Huberty (1994) states that. "A *hit* results when a unit emanating from group *g* is assigned (by means of the classification rule used) to group *g*" (p. 78). The hit rate is computed by dividing the hits by the total n of that group. When assessing the performance of variables as a subset, therefore, the researcher looks to the classification table which indicates hit rate for both groups, and or each group individually. An example of a classification table is found in Table 8. But stepwise methodology focuses on group separation, NOT hit rate. In fact, it should be noted that PDA prediction accuracy can actually go down with more predictors, unlike DDA where more response variables cannot hurt (increase) the value of Wilks's Λ (Huberty, 1984). DDA stepwise, therefore, is completely pointless when conducting DA for PDA classification purposes.

Insert Table 8 about here

Using the heuristic data from Holzinger and Swineford (1939), it was actually quite easy to find a set of predictors which yielded a better hit rate than the stepwise procedure. In addition, the results affected each group differently, which has practical implications as well. Table 9 contains the classification result from the stepwise output partially reproduced in Table 6. The total hit rate was found to be 69.8%. Substituting variable T9 for T17, the new total hit rate was higher: 70.4%. When one looks at the individual hit rates, the differences are even greater. For prediction in the eighth grade population, the hit rate went up almost 3 percentage points. This is important if the researcher is specifically interested in this population. This becomes especially important in high-stakes decision-making, like promotion or retention situations. I am sure that a potential eighth grader would feel more comfortable using the scores on the variables that I chose, than on those the computer stepwise program chose, if passing onto the eighth grade hung in the balance.

Insert Table 9 about here

Alternatives to Stepwise Methodology

When choosing the best subset of variables is important, a few alternatives are available, although not necessarily in computer packages such as SPSS. The first alternative, if you have the time, is to painstakingly substitute other variables into a discriminant analysis to arrive at possibly a better hit rate or a lower Wilks' Λ value, as seen in the heuristic data set.

Other alternatives are also available. Some specialized computer programs can also be used for purposes of variable selection. For example, for DDA cases the program offered by McCabe (1975) may be used. For PDA cases, the Morris program distributed within Huberty's (1994) book, or related programs by Morris described elsewhere (cf. Morris & Huberty, 1995). Huberty (1989) encouraged the use of these alternatives keeping the following in mind:

> (1) the collection of variables to be studied constitutes a meaningful system from a substantive standpoint; (2) the size of the initial variable system is manageable, that is, the number of response variables is about 30 or less; (3) there is a theoretical or substantive or practical reason for wanting to delete some variables; and (4) the researcher has access to a computer and some computer package software. (p. 49)

One alternative to stepwise which specifically looks at variable ordering considers using a partial F (i.e., considering the contribution of given variable partialling out the other variables)

(Huberty, 1989). Huberty goes on to outline other alternatives as well, including cross-validation procedures, acknowledging the need to utilize only those variables which fit theoretically, practically, and add to the predictive validity of our solution.

Some of the problems (i.e., wrong degrees of freedom) can be adjusted by hand, as noted in the previous sections. Just knowing that a problem exists leads to better alternatives and better analytical judgement. In conclusion, the bottom line seems to be that researchers need to learn how to think for themselves, and abstain from blind use of computer programs.

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Table 1

Output Discriminant Analysis

Discriminant

Group Statistics

TRACK			Valid N (listwise) Unweighted	Weighted
JUNE PROMOTIONS	T22		219	219.000
		T17	219	219.000
		T5	219	219.000
FEB PROMOTIONS		T22	82	82.000
		T17	82	82.000
		T5	82	82.000
Total	T22		301	301.000
		T17	301	301.000
		T5	301	301.000

Summary of Canonical Discriminant Functions

Eigenvalues

Function	Eigenvalue	% of Variance	Cumulative %	Canonical Correlation
1	.071(a)	100.0	100.0	.257

a. First 1 canonical discriminant functions were used in the analysis.

Wilks' Lambda

Test of Function(s)	Wilks' Lambda	Chi-square	df	Sig.
1	.934	20.315	3	.000

Standardized Canonical Discriminant Function Coefficients

	Function 1
T22	.265
T17	-.457
T5	.794

Structure Matrix

	Function 1
T5	.876
T22	.517
T17	-.367

Pooled within-groups correlations between discriminating variables and standardized canonical discriminant functions
Variables ordered by absolute size of correlation within function.

Functions at Group Centroids

TRACK	Function 1
JUNE PROMOTIONS	.162
FEB PROMOTIONS	-.433

Unstandardized canonical discriminant functions evaluated at group means

Classification Statistics

Prior Probabilities for Groups

TRACK	Prior	Cases Used in Analysis Unweighted	Weighted
JUNE PROMOTIONS	.728	219	219.000
FEB PROMOTIONS	.272	82	82.000
Total	1.000	301	301.000

Classification Results(a)

		Predicted Group Membership TRACK	JUNE PROMOTIONS	FEB PROMOTIONS	Total
Original	Count	JUNE PROMOTIONS	211	8	219
		FEB PROMOTIONS	74	8	82
	%	JUNE PROMOTIONS	96.3	3.7	100.0
		FEB PROMOTIONS	90.2	9.8	100.0

a 72.8% of original grouped cases correctly classified.

Table 2

Posterior Probabilities

Illustrative Results for the First 50 Cases

Case Number	Mis Val	Sel	Actual Group	Highest Probability Group	P(D/G)	P(G/D)	2nd Highest Group	P(G/D)	Discrim Scores
1			2 **	1	.8160	.5782	2	.4218	.3948
2			2	2	.6146	.6170	1	.3830	-.9365
3			2	2	.6270	.6145	1	.3855	-.9190
4			2 **	1	.9137	.5601	2	.4399	.2704
5			2	2	.5196	.6365	1	.3635	-1.0770
6			2	2	.7741	.5861	1	.4139	-.7201
7			2 **	1	.9863	.5416	2	.4584	.1449
8			2	2	.4778	.6456	1	.3544	-1.1428
9			2	2	.1393	.7421	1	.2579	-1.9113
10			2	2	.8421	.5146	1	.4854	-.2338
11			2	2	.9846	.5470	1	.4530	-.4523
12			2 **	1	.7752	.5018	2	.4982	-.1234
13			2	2	.8670	.5688	1	.4312	-.6005
14			2	2	.8202	.5775	1	.4225	-.6603
15			2 **	1	.2128	.7147	2	.2853	1.4080
16			2	2	.9005	.5626	1	.4374	-.5581
17			2	2	.4988	.6410	1	.3590	-1.1093
18			2	2	.6665	.6067	1	.3933	-.8640
19			2 **	1	.0195	.8274	2	.1726	2.4981
20			2 **	1	.6116	.6176	2	.3824	.6700
21			2	2	.3283	.6811	1	.3189	-1.4106
22			2	2	.4797	.6451	1	.3549	-1.1397
23			2	2	.9471	.5344	1	.4656	-.3667
24			2	2	.9266	.5305	1	.4695	-.3409
25			2 **	1	.9465	.5342	2	.4658	.0950
26			2 **	1	.7367	.5932	2	.4068	.4983
27			2	2	.1422	.7409	1	.2591	-1.9007
28			2	2	.5392	.6324	1	.3676	-1.0470
29			2	2	.5869	.6226	1	.3774	-.9764
30			2	2	.7417	.5922	1	.4078	-.7626
31			2	2	.7013	.6000	1	.4000	-.8165
32			2	2	.9355	.5561	1	.4439	-.5139
33			2 **	1	.2551	.7015	2	.2985	1.3003
34			2	2	.8872	.5232	1	.4768	-.2912
35			2	2	.4792	.6453	1	.3547	-1.1407
36			2	2	.8387	.5140	1	.4860	-.2295
37			2	2	.9841	.5412	1	.4588	-.4131
38			2	2	.7761	.5020	1	.4980	-.1486
39			2	2	.8923	.5241	1	.4759	-.2975
40			2 **	1	.0576	.7870	2	.2130	2.0607
41			2	2	.6082	.6183	1	.3817	-.9457
42			1 **	2	.7666	.5001	1	.4999	-.1362
43			1 **	2	.9711	.5495	1	.4505	-.4692
44			1 **	2	.6646	.6071	1	.3929	-.8666
45			1	1	.7664	.5876	2	.4124	.4592
46			1 **	2	.7029	.5997	1	.4003	-.8144
47			1 **	2	.6761	.6048	1	.3952	-.8507
48			1 **	2	.6305	.6138	1	.3862	-.9140
49			1 **	2	.9528	.5529	1	.4471	-.4922
50			1	1	.6456	.6108	2	.3892	.6220

Table 3

Classification Table

Classification Results(a)

		TRACK	Predicted Group Membership JUNE PROMOTIONS	FEB PROMOTIONS	Total
Original	Count	JUNE PROMOTIONS	211	8	219
		FEB PROMOTIONS	74	8	82
	%	JUNE PROMOTIONS	96.3	3.7	100.0
		FEB PROMOTIONS	90.2	9.8	100.0

a 72.8% of original grouped cases correctly classified.

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Table 4

Standardized Canonical Discriminant Function Coefficients

Standardized Canonical Discriminant Function Coefficients

	Function 1
T22	.265
T17	-.457
T5	.794

Table 5

Output Table from Stepwise Discriminant Analysis

Step	Variable	Wilks' Λ	F-statistic	df	p-value
1	T10	0.878	41.360	1	0.000
2	T19	0.839	28.676	2	0.000
3	T3	0.815	22.539	3	0.000
4	T17	**0.805**	17.948	4	0.000

Table 6

Discriminant Analysis Output Substituting T12 for T17

Function	Wilks' Λ	Chi-square	df	p-value
1	**0.805**	64.338	4	0.000

Table 7

Correcting for the Wrong Degrees of Freedom

$F = [(1 - \Lambda) / \Lambda]* (\text{df error} / \text{df effect})$

STEP ONE: [(1-0.878) / 0.878] * (298 / 1)

[0.122 / 0.878] * 298

[.01389522] * 298

41.408

STEP ONE: [(1-0.878) / 0.878] * (277 / 21)

[0.122 / 0.878] * 13.19

[.01389522] * 13.19

0.0105347

Table 8

Classification Table for Stepwise Output

		Predicted Group	
		7	8
Original Group	7	118 (75.2%)	39 (24.8%)
	8	52 (36.1%)	92 (63.9%)

69.8% of original grouped cases correctly classified

Table 9

Classification Table Substituting T9 for T17

		Predicted Group	
		7	8
Original Group	7	117 (74.5%)	40 (25.5%)
	8	49 (34%)	95 (66%)

70.4% of original grouped cases correctly classified

Appendix A

```
SET BLANKS=SYSMIS UNDEFINED=WARN printback=list.
TITLE 'MULTINOR.SPS   tests multivar normality graphically****'.
COMMENT *****************************************************.
COMMENT The original MULTINOR computer program was presented,
COMMENT with examples, in:
COMMENT     Thompson, B. (1990). MULTINOR: A FORTRAN program that
COMMENT     assists in evaluating multivariate normality.
COMMENT     _Educational and Psychological Measurement_, 50,
COMMENT     845-848.
COMMENT
COMMENT The logic and the data source for the example are from:
COMMENT     Stevens, J. (1986). _Applied multivariate statistics
COMMENT     for the social sciences_. Hillsdale, NJ: Erlbaum.
COMMENT     (pp. 207-212)
COMMENT *******************************************************.
COMMENT  Here there are 3 variables for which multivariate
COMMENT  normality is being confirmed.
DATA LIST
  FILE='c:\spsswin\multinor.dat' FIXED RECORDS=1 TABLE
  /1 x1 1-3 (1) x2 5-7 (1) x3 9-11 (1).
list variables=all/cases=9999/format=numbered .
COMMENT  'y' is a variable automatically created by the program, and
COMMENT  does not have to modified for different data sets.
compute y=$casenum .
print formats y(F5) .
regression variables=y x1 to x3/
  descriptive=mean stddev corr/
  dependent=y/enter x1 to x3/
  save=mahal(mahal) .
sort cases by mahal(a) .
execute .
list variables=y x1 to x3 mahal/cases=9999/format=numbered .
COMMENT  In the next TWO lines, for a given data set put the actual n
COMMENT  in place of the number '12' used for the example data set.
loop #i=1 to 12 .
COMMENT  In the next line, change '3' to whatever is the number
COMMENT  of variables.
COMMENT       The p critical value of chi square for a given case
COMMENT  is set as [the case number (after sorting) - .5] / the
COMMENT  sample size].
compute p=($casenum - .5) / 12. .
compute chisq=idf.chisq(p,3) .
end loop .
print formats p chisq (F8.5) .
list variables=y p mahal chisq/cases=9999/format=numbered .
plot
  vertical='chi square'/
  horizontal='Mahalabis distance'/
  plot=chisq with mahal .
```

Appendix B

```
  FILE='c:\program files\spss\coursework\HOLZINGR.DTA' FIXED RECORDS=2 TABLE
  /1 id 1-3 sex 4-4 ageyr 6-7
  agemo 8-9 t1 11-12 t2 14-15 t3 17-18 t4 20-21 t5 23-24 t6 26-27 t7 29-30 t8
  32-33 t9 35-36 t10 38-40 t11 42-44 t12 46-48 t13 50-52 t14 54-56 t15 58-60
  t16 62-64 t17 66-67 t18 69-70 t19 72-73 t20 74-76 t21 78-79 /2 t22 11-12
  t23 14-15 t24 17-18 t25 20-21 t26 23-24 .
EXECUTE.
COMPUTE SCHOOL=1.
IF (ID GT 200)SCHOOL=2.
IF (ID GE 1 AND ID LE 85)GRADE=7.
IF (ID GE 86 AND ID LE 168)GRADE=8.
IF (ID GE 201 AND ID LE 281)GRADE=7.
IF (ID GE 282 AND ID LE 351)GRADE=8.
IF (ID GE 1 AND ID LE 44)TRACK=2.
IF (ID GE 45 AND ID LE 85)TRACK=1.
IF (ID GE 86 AND ID LE 129)TRACK=2.
IF (ID GE 130)TRACK=1.
PRINT FORMATS SCHOOL TO TRACK(F1.0).
VALUE LABELS SCHOOL(1)PASTEUR (2) GRANT-WHITE/
  TRACK (1)JUNE PROMOTIONS (2)FEB PROMOTIONS/.
VARIABLE LABELS T1 VISUAL PERCEPTION TEST FROM SPEARMAN VPT, PART III
  T2 CUBES, SIMPLIFICATION OF BRIGHAM'S SPATIAL RELATIONS TEST
  T3 PAPER FORM BOARD--SHAPES THAT CAN BE COMBINED TO FORM A TARGET
  T4 LOZENGES FROM THORNDIKE--SHAPES FLIPPED OVER THEN IDENTIFY TARGET
  T5 GENERAL INFORMATION VERBAL TEST
  T6 PARAGRAPH COMPREHENSION TEST
  T7 SENTENCE COMPLETION TEST
  T8 WORD CLASSIFICATION--WHICH WORD NOT BELONG IN SET
  T9 WORD MEANING TEST
  T10 SPEEDED ADDITION TEST
  T11 SPEEDED CODE TEST--TRANSFORM SHAPES INTO ALPHA WITH CODE
  T12 SPEEDED COUNTING OF DOTS IN SHAPE
  T13 SPEEDED DISCRIM STRAIGHT AND CURVED CAPS
  T14 MEMORY OF TARGET WORDS
  T15 MEMORY OF TARGET NUMBERS
  T16 MEMORY OF TARGET SHAPES
  T17 MEMORY OF OBJECT-NUMBER ASSOCIATION TARGETS
  T18 MEMORY OF NUMBER-OBJECT ASSOCIATION TARGETS
  T19 MEMORY OF FIGURE-WORD ASSOCIATION TARGETS
  T20 DEDUCTIVE MATH ABILITY
  T21 MATH NUMBER PUZZLES
  T22 MATH WORD PROBLEM REASONING
  T23 COMPLETION OF A MATH NUMBER SERIES
  T24 WOODY-MCCALL MIXED MATH FUNDAMENTALS TEST
  T25 REVISION OF T3--PAPER FORM BOARD
  T26 FLAGS--POSSIBLE SUBSTITUTE FOR T4 LOZENGES.

DISCRIMINANT
  /GROUPS=grade(7 8)
  /VARIABLES=t1 t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t2 t20 t21 t22 t23
  t24 t25 t26 t3 t4 t5 t6 t7 t8 t9
  /ANALYSIS ALL
  /PRIORS  SIZE
  /STATISTICS=RAW CORR COV GCOV CROSSVALID
  /CLASSIFY=NONMISSING POOLED .
```

```
DISCRIMINANT
  /GROUPS=grade(7 8)
  /VARIABLES=t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21 t22 t23 t24 t3 t4
    t5 t6 t7 t8 t9
  /ANALYSIS ALL
   /METHOD=WILKS
   /FIN= 3.5
   /FOUT= 2.71
  /PRIORS  SIZE
   /HISTORY
  /STATISTICS=TABLE
  /CLASSIFY=NONMISSING POOLED .

DISCRIMINANT
  /GROUPS=grade(7 8)
  /VARIABLES=t3 t10 t12 t19
  /ANALYSIS ALL
  /PRIORS  SIZE
  /STATISTICS=TABLE
  /CLASSIFY=NONMISSING POOLED .
```

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Printed Name: Tanya Woldbeck	Organization: Texas A&M University
Address: TAMU Dept Educ Psyc College Station, TX 77843-4225	Telephone Number: (409) 845-1335
	Date: 3/16/98